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Adjacency Eigenvalues for Underlying Split Multigraphs

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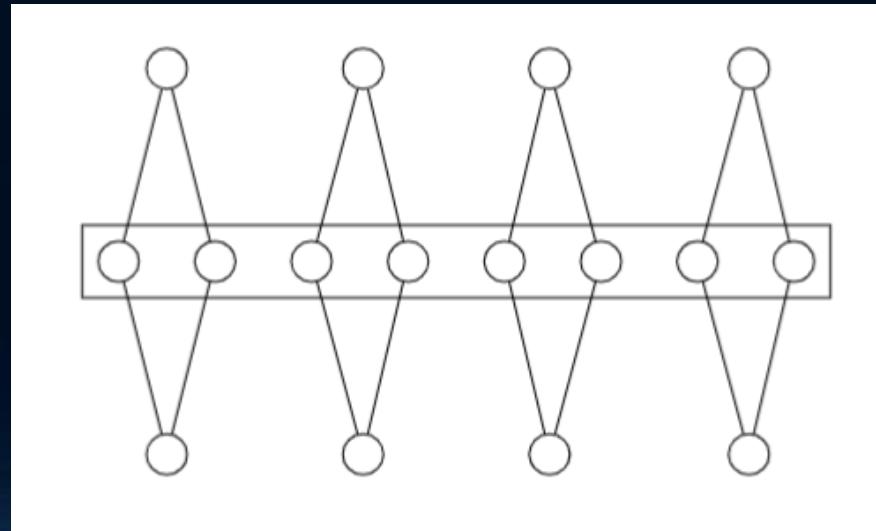DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE
MENTOR: DR. SACCOMAN

Background Information

$$x - IPS(c, d)^{\mu}$$

- c = number of cones

- d = degree of each cone

- μ = multiplicity of edges within clique

- x = number of cone nodes to which each clique node is adjacent

$$2 - IPS(8, 2)^{\mu}$$

Research Question: If these multigraphs represent satellite and ground station communication, can we find a formula to best represent the number of triangles from the ground station to the satellite?

Method

Adjacency Matrix

$$3 - IPS(9,3)^1$$

Adjacency Matrix

Characteristic Polynomial

- Submatrix:

CharacteristicPolynomial[CP91, x]

$8 + 63 x + 216 x^2 + 420 x^3 + 504 x^4 + 378 x^5 + 168 x^6 + 36 x^7 - x^9$

168 / 2

84

- Whole Matrix:

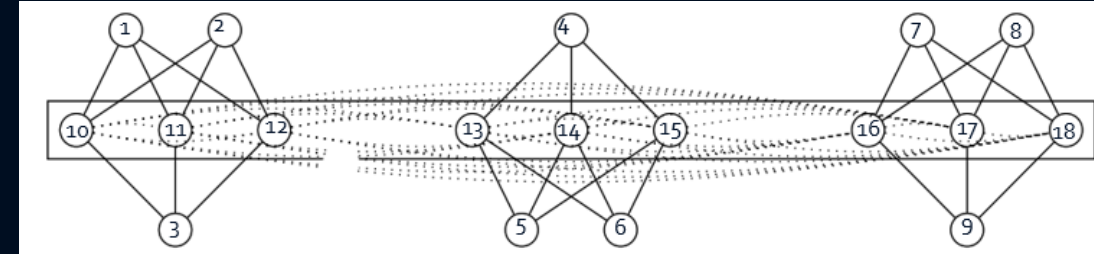CharacteristicPolynomial[IPS931, x]

$- 729 x^6 - 4860 x^7 - 13473 x^8 - 19502 x^9 - 14427 x^{10} - 2862 x^{11} + 3360 x^{12} + 2358 x^{13} + 243 x^{14} - 222 x^{15} - 63 x^{16} + x^{18}$
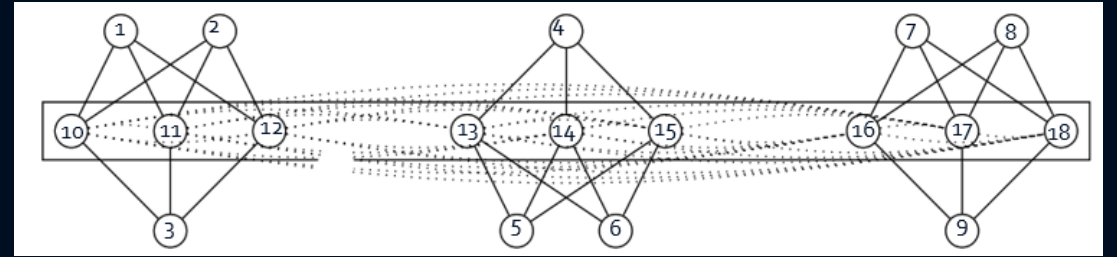
222 / 2

111

111 – 84

27

Conjecture

1. *Number of triangles can be found with the formula:* $\frac{(x-1)}{2} * (x^2 * c_o * \mu)$

2. $c_o(x-1) = $ *number of eigenvalues equalling zero*

3. $c_o(x-1) = $ *number of eigenvalues equalling* $-\mu$

4. *There are two eigenvalues that are the roots of* $\mathrm{x}^2 + \mu\mathrm{x} + -(x^2)$

5. *There is one eigenvalue that is the roots of* $\mathrm{x}^2 + (-\mu)(c - 1)\mathrm{x} - x^2$
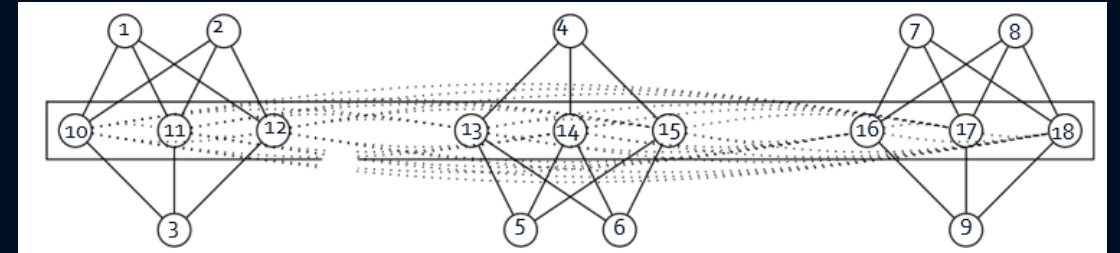
Triangles

- Conjecture 1: $\frac{(x-1)}{2} * (x^2 * c_o * \mu)$

$$\frac{(3-1)}{2} * (3^2 * 3 * 1)$$
$$1 * (9 * 1)$$
$$\mathbf{27}$$

$$* Note: c = x * c_o$$
$$9 = 3 * 3$$

Eigenvalues

$$\{9., -3.54138, -3.54138, 2.54138, 2.54138, -1., \boxed{-1., -1., -1., -1., -1., -1., 0., 0., 0., 0., 0., 0.}\}$$

Eigenvalues = 0 have a multiplicity of 6
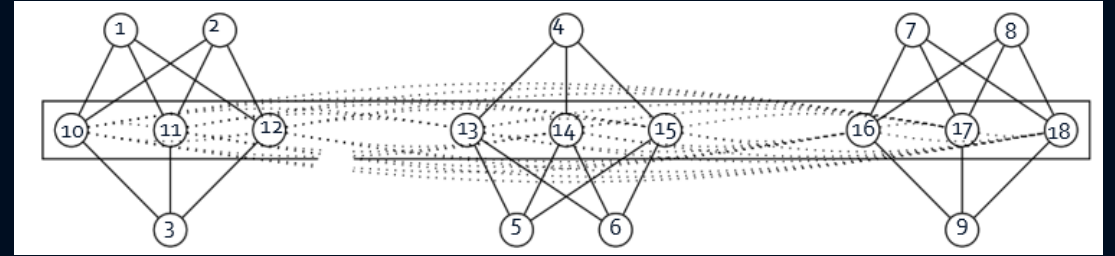Eigenvalues = -1 have a multiplicity of 6

$$3 - IPS(9,3)^1$$
$$c_o = 3$$
$$x = 3$$

Conjecture 2 & 3: $c_o(x-1) = number\ of\ eigenvalues\ equalling\ zero\ \&\ equalling\ -\mu$

Eigenvalues

- Conjecture 4:
  There are two eigenvalues that are the roots of $x^2 + \mu x + -(x^2)$

$$\{9., -3.54138, -3.54138, 2.54138, 2.54138, -1., -1., -1., -1., -1., -1., -1., 0., 0., 0., 0., 0., 0.\}$$
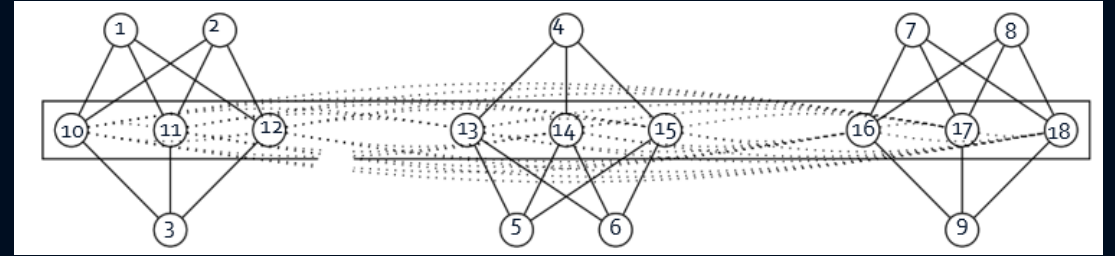
$x^2 + \mu x + (-x^2)$
$1x^2 + 1x - 9$

Factored Polynomial Quotient: $(x^2 + x - 9)$

A = 1

B = 1

C = -9

$$\frac{\left(B \pm \sqrt{B^2 - 4AC}\right)}{2A}$$

-3.54138, 2.54138

Eigenvalues

- Conjecture 5: *There is one eigenvalue that is the roots of* $x^2 + (-\mu)(c-1)x - x^2$

$$\{9., -3.54138, -3.54138, 2.54138, 2.54138, -1., -1., -1., -1., -1., -1., -1., 0., 0., 0., 0., 0., 0.\}$$

$x^2 + (-\mu)(c-1)x - x^2$

$1x^2 + (-1)(9-1)(1)x - 3^2$
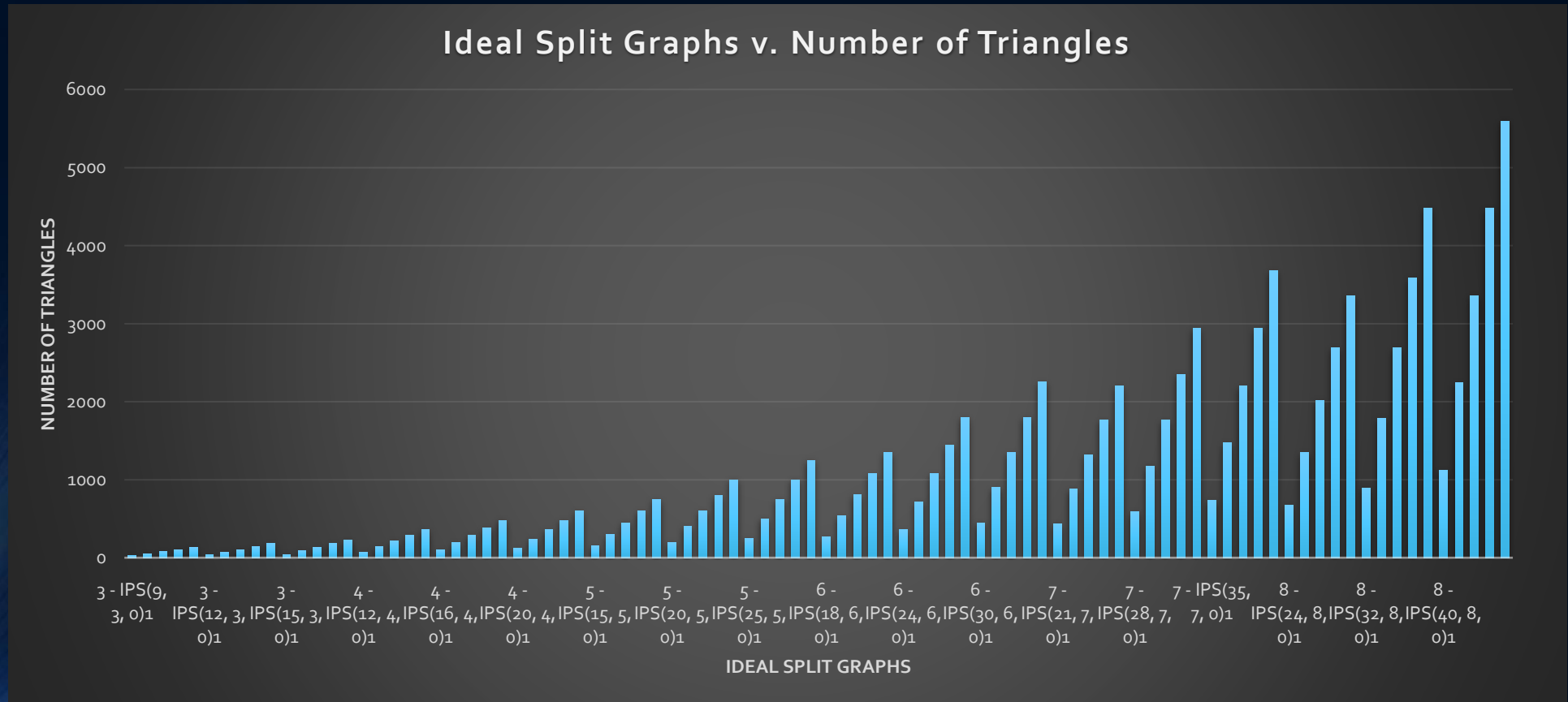
$x^2 - 8x - 9$

Irreducible: $(x^2 - 8x - 9)$

A = 1

B = -8

C = -9

$\dfrac{\left(B \pm \sqrt{B^2 - 4AC}\right)}{2A}$

9, -1

Data

Thank you! Questions?