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Generating Variance Analysis Models Based on Responsibility Centers

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by

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1. INTRODUCTION

A ubiquitous Accounting tool for tracking various business performance metrics (cost, efficiency, productivity, profit, revenue, etc.), variance analysis is the study of the difference between budgeted (also known as planned or standard) and actual numbers in a single period. Its application can be commonly found in disciplines including Marketing[1] and Operations Management[2], and in various industries such as air travel[3], construction[4], health care[5], and hospitality[6]. This document presents an algorithm for generating variance analysis models based on responsibility centers (i.e., accountable decision makers or managers). Further discussion of this approach can be found in publication.[7]

2. MODEL GENERATION

We build our models based on two generally-held premises of budget variance analysis. First, "...it is important to designate the portion of the total variance that is attributable to each manager."[8] In other words, models should be a function of the number and combination of responsibility centers. To account for the possibility that for an n-variable product there may be any of $0, 1, 2, \ldots, n$ responsibility centers, $n+1$ general-form models (i.e., one for zero responsibility centers, one for one responsibility center, etc.) can be generated. More specifically, 2^n models can be generated accounting for all possible combinations of n variables, each of which is or is not attributed to a responsibility center. The zero- and n-responsibility center models are the same. Therefore, for an n-variable product there are $2^n - 1$ unique, final models. We are unaware of any existing models derived explicitly from this premise. Second, a flexible budget is an estimate of what revenue and costs should have been, had the manager(s) known the actual value of variables not assigned to their respective responsibility center(s) in advance. Robert Kaplan, a leading expert on the topic of variance analysis, states, "... managers expect that many of their indirect and support expenses should be managed or controlled based on *actual* activity levels during the period"[9] We refer to this as "Kaplan's premise." Given this, our models are generated such that the variance of a variable that is attributed to a responsibility center will be calculated using the *actual* value of the variable(s) not attributed to a responsibility center. A corollary is that the variance of a variable that is not attributed to a responsibility center will be calculated using the budgeted value of the variable(s) that is (are) attributed to a responsibility center. Note that there may be cases where the latter premise is debatable, such as demand being a function of price. It may not be reasonable to expect a sales department to meet its respective budget of units sold if selling price increases. That said, we adopt Kaplan's premise, acknowledge circumstances under which it may not be appropriate and address the concern in separate work.

In generating the most commonly-used revenue variance model, quantity variance is calculated using the budgeted value of price. The quantity variable is then "flexed" from its budgeted to actual value to calculate price variance. (Considering Kaplan's premise, this suggests that there is a responsibility center for setting unit price, but not one for sales.) Thus, with a two-variable product, there is one flexing step. It is easy to show that for an n-variable product there will be $n - 1$ flexing steps. In generating our models, we follow the general procedure of calculating a variance, flexing the respective variable, calculating another variance, flexing the respective variable, etc., until all variances have been calculated.

Note that for a two-variable product with one responsibility center there will be only one model that satisfies the above requirements. For all other cases of $n \geq 2$, there will be multiple possible models resulting from the different orders in which the variances are calculated and variables are flexed. In these cases the final model is calculated as the average of all the possible applicable models. This addresses an outstanding issue related to the order in which variables are flexed to generate a model.[10] We assume no preference for the order in which variables are flexed, other than all those not assigned to a responsibility center are flexed before all those that are.

3. ALGORITHM FOR GENERATING MODELS

Here we provide our algorithm used to generate the budget variance analysis models for an n-variable product. Generally, the algorithm begins with all variables set to their budgeted values. It progressively alters each variable (those of non-responsibility centers first) to its actual value once the respective variance has been calculated. Variables are indicated by lower-case letters. Where applicable, a variable's respective responsibility center is indicated by the same upper-case letter. Each variable can take on two values, actual and budgeted (denoted by subscripts a and b, respectively). The difference of any variable z is calculated as $\Delta z = z_a - z_b$. The average of any variable z is calculated as $\bar{z} = (z_a + z_b)/2$. The algorithm is illustrated by the bulleted items using a three-variable product $v(w, x, y) = wxy$, where w is a variable attributed to Responsibility Center W, and x and y are variables that are not attributed to a responsibility center.

1. Begin a new intermediate model with all variables taking on their respective budgeted values.

   - w_b, x_b and y_b.

2. Select a variable that is not attributed to a responsibility center. If there are none, select a remaining variable.

   - Select x.

3. For the selected variable, calculate its variance as its difference multiplied by the product of the budgeted values of the remaining $n-1$ variables.
   - Calculate x variance as $(x_a - x_b)w_b y_b = \Delta x w_b y_b$.
4. Update the value of the selected variable to its actual value, to be used in subsequent calculations.
   - x_b is updated to x_a, w_b and y_b remain unchanged.
5. Select a variable that is not attributed to a responsibility center whose variance has not yet been calculated. If there are none, select a remaining variable for which a variance has not yet been calculated.
   - Select y.
6. For this newly-selected variable, calculate its variance as its difference multiplied by the product of the values of the remaining $n-1$ variables.
   - Calculate y variance as $(y_a - y_b)w_b x_a = \Delta y w_b x_a$.
7. Update the value of the newly-selected variable to its actual value, to be used in subsequent calculations.
   - y_b is updated to y_a, w_b and x_a remain unchanged.
8. Repeat steps 5 through 7 until a variance has been calculated for each variable. This set of variances comprises the given intermediate model.
   - Step 5 (second pass): select w.
   - Step 6 (second pass): w variance $= \Delta w x_a y_a$
   - Step 7 (second pass): w_b is updated to w_a, x_a and y_a remain unchanged.
   - Intermediate Model A: x variance $= \Delta x w_b y_b$, y variance $= \Delta y w_b x_a$ and w variance $= \Delta w x_a y_a$.

9. Repeat steps 1 through 8 until all intermediate models have been generated.

- Step 1: w_b, x_b and y_b.
- Step 2: Select y.
- Step 3: Calculate y variance as $(y_a - y_b)w_b x_b = \Delta y w_b x_b$.
- Step 4: y_b is updated to y_a, w_b and x_b remain unchanged.
- Step 5: Select x.
- Step 6: x variance $= \Delta x w_b y_a$
- Step 7: x_b is updated to x_a, w_b and y_a remain unchanged.
- Step 8:
  - Step 5 (second pass): Select w.
  - Step 6 (second pass): w variance $= \Delta w x_a y_a$
  - Step 7 (second pass): w_b is updated to w_a, x_a and y_a remain unchanged.
  - Intermediate Model B: x variance $= \Delta x w_b y_b$, y variance $= \Delta y w_b x_a$ and w variance $= \Delta w x_a y_a$.

Variable	**Intermediate Model**	
	A	**B**
w	$\Delta w x_a y_a$	$\Delta w x_a y_a$
x	$\Delta x w_b y_b$	$\Delta x w_b y_a$
y	$\Delta y w_b x_a$	$\Delta y w_b x_b$

10. For each variable, calculate its variance as the average of its respective variances from the intermediate models. The set of the variances comprises the final model.

Variable	Model
w	$\dfrac{\Delta w x_a y_a + \Delta w x_a y_a}{2} = \Delta w \dfrac{2 x_a y_a}{2} = \Delta w x_a y_a$
x	$\dfrac{\Delta x y_b w_b + \Delta x y_a w_b}{2} = \Delta x w_b \dfrac{y_b + y_a}{2} = \Delta x w_b \bar{y}$
y	$\dfrac{\Delta y x_a w_b + \Delta y x_b w_b}{2} = \Delta y w_b \dfrac{x_a + x_b}{2} = \Delta y w_b \bar{x}$

4. RESULTS

Here we provide the models for $n = 2$, 3 and 4 variables, generated using the algorithm above. See Tables 1, 2 and 3.

TABLE 1
Variance Analysis Models for a Two-Variable Product

Variable	Responsibility Center(s)	
	W	**W and X***
w	$\Delta w x_a$	$\Delta w \bar{x}$
x	$\Delta x w_b$	$\Delta x \bar{w}$
$v(w, x)$	$w_a x_a - w_b x_b$	

*Also applicable for the case of no responsibility centers.

TABLE 2
Variance Analysis Models for a Three-Variable Product

Variable	Responsibility Center(s)		
	W	**W and X**	**W, X and Y***
w	$\Delta w x_a y_a$	$\Delta w \bar{x} y_a$	$\Delta w \left(\bar{x}\bar{y} + \frac{\Delta x \Delta y}{12} \right)$
x	$\Delta x w_b \bar{y}$	$\Delta x \bar{w} y_a$	$\Delta x \left(\bar{w}\bar{y} + \frac{\Delta w \Delta y}{12} \right)$
y	$\Delta y w_b \bar{x}$	$\Delta y w_b x_b$	$\Delta y \left(\bar{w}\bar{x} + \frac{\Delta w \Delta x}{12} \right)$
$v(w, x, y)$	$w_a x_a y_a - w_b x_b y_b$		

*Also applicable for the case of no responsibility centers.

TABLE 3
Variance Analysis Models for a Four-Variable Product

Variable	**Responsibility Center(s)**			
	W	**W and X**	**W, X and Y**	**W, X, Y and Z***
w	$\Delta w x_a y_a z_a$	$\Delta w \bar{x} y_a z_a$	$\Delta w z_a \left(\bar{x}\bar{y} + \frac{\Delta x \Delta y}{12} \right)$	$\Delta w \left(\bar{x}\bar{y}\bar{z} + \frac{\Delta x \Delta y \bar{z} + \Delta x \bar{y} \Delta z + \bar{x} \Delta y \Delta z}{12} \right)$
x	$\Delta x w_b \left(\bar{y}\bar{z} + \frac{\Delta y \Delta z}{12} \right)$	$\Delta x \bar{w} y_a z_a$	$\Delta x z_a \left(\bar{w}\bar{y} + \frac{\Delta w \Delta y}{12} \right)$	$\Delta x \left(\bar{w}\bar{y}\bar{z} + \frac{\Delta w \Delta y \bar{z} + \Delta w \bar{y} \Delta z + \bar{w} \Delta y \Delta z}{12} \right)$
y	$\Delta y w_b \left(\bar{x}\bar{z} + \frac{\Delta x \Delta z}{12} \right)$	$\Delta y w_b x_b \bar{z}$	$\Delta y z_a \left(\bar{w}\bar{x} + \frac{\Delta w \Delta x}{12} \right)$	$\Delta y \left(\bar{w}\bar{x}\bar{z} + \frac{\Delta w \Delta x \bar{z} + \Delta w \bar{x} \Delta z + \bar{w} \Delta x \Delta z}{12} \right)$
z	$\Delta z w_b \left(\bar{x}\bar{y} + \frac{\Delta x \Delta y}{12} \right)$	$\Delta z w_b x_b \bar{y}$	$\Delta z w_b x_b y_b$	$\Delta z \left(\bar{w}\bar{x}\bar{y} + \frac{\Delta w \Delta x \bar{y} + \Delta w \bar{x} \Delta y + \bar{w} \Delta x \Delta y}{12} \right)$
$v(w,x,y,z)$	$w_a x_a y_a z_a - w_b x_b y_b z_b$			

*Also applicable for the case of no responsibility centers.

5. EXAMPLE

Here we provide a numerical example using two products. For each product, profit is calculated as total revenue (two-variable) minus total spending (three-variable direct materials plus four-variable direct labor), with unit cost c, labor rate l, unit price p, quantity sold q, production time t, usage u and exchange rate x: $\pi(c, l, p, q, t, u, x) = pq - (cuq + ltqx)$. We use the same exchange rate for the two products. Each variable is assigned to a responsibility center and therefore we use the right-most column in each of Tables 1, 2 and 3 for each variable's respective variance. We begin with the model for a single product. See Table 4.

TABLE 4
Variance Analysis Model for Single-Product Profit

Variable	**Variance**
Unit Price, p	$\Delta p \bar{q}$
Unit Cost, c	$-\Delta c \left(\bar{q}\bar{u} + \frac{\Delta q \Delta u}{12} \right)$
Usage, u	$-\Delta u \left(\bar{c}\bar{q} + \frac{\Delta c \Delta q}{12} \right)$
Labor Rate, l	$-\Delta l \left(\bar{t}\bar{q}\bar{x} + \frac{\Delta t \Delta q \bar{x} + \Delta t \bar{q} \Delta x + \bar{t} \Delta q \Delta x}{12} \right)$
Production Time, t	$-\Delta t \left(\bar{l}\bar{q}\bar{x} + \frac{\Delta l \Delta q \bar{x} + \Delta l \bar{q} \Delta x + \bar{l} \Delta q \Delta x}{12} \right)$
Exchange Rate, x	$-\Delta x \left(\bar{l}\bar{t}\bar{q} + \frac{\Delta l \Delta t \bar{q} + \Delta l \bar{t} \Delta q + \bar{l} \Delta t \Delta q}{12} \right)$
Quantity Sold, q	$\Delta q \left(\bar{p} - \left(\bar{c}\bar{u} + \frac{\Delta c \Delta u}{12} + \bar{l}\bar{t}\bar{x} + \frac{\Delta l \Delta t \bar{x} + \Delta l \bar{t} \Delta x + \bar{l} \Delta t \Delta x}{12} \right) \right)$

Note that as quantity sold appears in three terms in the profit function, its variance incorporates all the other variables in those three terms. For variables that appear in the profit function following a negative sign (unit cost, usage, etc.), the respective variance calculation is given a negative sign as well. This corresponds to the variance being unfavorable if the variable's actual value is greater than its budgeted value.

Next, we incorporate the second product. Subscripts 1 and 2 denote the respective product. See Table 5.

Note that since exchange rate is common to the direct labor spending of both products, its variance incorporates labor rate, production time and quantity sold of both products.

To complete the numerical example, see Table 6. Shown are the values of the parameters and the resulting variances found using the model in Table 5.

TABLE 5
Variance Analysis Model for Two-Product Profit

Variable	Variance
Unit Price 1, p_1	$\Delta p_1 \bar{q}_1$
Unit Cost 1, c_1	$-\Delta c_1 \left(\bar{q}_1 \bar{u}_1 + \frac{\Delta q_1 \Delta u_1}{12} \right)$
Usage 1, u_1	$-\Delta u_1 \left(\bar{c}_1 \bar{q}_1 + \frac{\Delta c_1 \Delta q_1}{12} \right)$
Labor Rate 1, l_1	$-\Delta l_1 \left(\bar{t}_1 \bar{q}_1 \bar{x} + \frac{\Delta t_1 \Delta q_1 \bar{x} + \Delta t_1 \bar{q}_1 \Delta x + \bar{t}_1 \Delta q_1 \Delta x}{12} \right)$
Production Time 1, t_1	$-\Delta t_1 \left(\bar{l}_1 \bar{q}_1 \bar{x} + \frac{\Delta l_1 \Delta q_1 \bar{x} + \Delta l_1 \bar{q}_1 \Delta x + \bar{l}_1 \Delta q_1 \Delta x}{12} \right)$
Quantity Sold 1, q_1	$\Delta q_1 \left(\bar{p}_1 - \left(\bar{c}_1 \bar{u}_1 + \frac{\Delta c_1 \Delta u_1}{12} + \bar{l}_1 \bar{t}_1 \bar{x} + \frac{\Delta l_1 \Delta t_1 \bar{x} + \Delta l_1 \bar{t}_1 \Delta x + \bar{l}_1 \Delta t_1 \Delta x}{12} \right) \right)$
Unit Price 2, p_2	$\Delta p_2 \bar{q}_2$
Unit Cost 2, c_2	$-\Delta c_2 \left(\bar{q}_2 \bar{u}_2 + \frac{\Delta q_2 \Delta u_2}{12} \right)$
Usage 2, u_2	$-\Delta u_2 \left(\bar{c}_2 \bar{q}_2 + \frac{\Delta c_2 \Delta q_2}{12} \right)$
Labor Rate 2, l_2	$-\Delta l_2 \left(\bar{t}_2 \bar{q}_2 \bar{x} + \frac{\Delta t_2 \Delta q_2 \bar{x} + \Delta t_2 \bar{q}_2 \Delta x + \bar{t}_2 \Delta q_2 \Delta x}{12} \right)$
Production Time 2, t_2	$-\Delta t_2 \left(\bar{l}_2 \bar{q}_2 \bar{x} + \frac{\Delta l_2 \Delta q_2 \bar{x} + \Delta l_2 \bar{q}_2 \Delta x + \bar{l}_2 \Delta q_2 \Delta x}{12} \right)$
Quantity Sold 2, q_2	$\Delta q_2 \left(\bar{p}_2 - \left(\bar{c}_2 \bar{u}_2 + \frac{\Delta c_2 \Delta u_2}{12} + \bar{l}_2 \bar{t}_2 \bar{x} + \frac{\Delta l_2 \Delta t_2 \bar{x} + \Delta l_2 \bar{t}_2 \Delta x + \bar{l}_2 \Delta t_2 \Delta x}{12} \right) \right)$
Exchange Rate, x	$-\Delta x \left(\bar{l}_1 \bar{t}_1 \bar{q}_1 + \frac{\Delta l_1 \Delta t_1 \bar{q}_1 + \Delta l_1 \bar{t}_1 \Delta q_1 + \bar{l}_1 \Delta t_1 \Delta q_1}{12} + \bar{l}_2 \bar{t}_2 \bar{q}_2 + \frac{\Delta l_2 \Delta t_2 \bar{q}_2 + \Delta l_2 \bar{t}_2 \Delta q_2 + \bar{l}_2 \Delta t_2 \Delta q_2}{12} \right)$

TABLE 6
Variance Analysis Parameters and Results for Two-Product Profit

Variable	**Budgeted**	**Actual**	**Variance**
Unit Price 1, p_1	50.00	55.00	\$562.50
Unit Cost 1, c_1	1.00	1.10	(\$21.92)
Usage 1, u_1	2.00	1.90	\$11.83
Labor Rate 1, l_1	1.00	1.20	(\$134.53)
Production Time 1, t_1	2.00	1.80	\$78.48
Quantity Sold 1, q_1	100	125	\$1096.91
Unit Price 2, p_2	75.00	70.00	(\$950.00)
Unit Cost 2, c_2	2.00	1.90	\$58.93
Usage 2, u_2	3.00	3.20	(\$74.07)
Labor Rate 2, l_2	4.00	3.70	\$935.64
Production Time 2, t_2	5.00	5.40	(\$921.26)
Quantity Sold 2, q_2	180	200	\$68.51
Exchange Rate, x	3.00	3.30	(\$1211.09)

Notes

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