

# An Exploration of graphs that provide a lower bound on the number of spanning trees

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## Abstract:

We denote by  $L(n, k, c)$  the graphs on  $n$  nodes having a clique of order  $k$ , and a non-clique node adjacent to  $c$  of the clique nodes with a path on  $n - (k + 1)$  nodes appended to it. While it has been proven that these graphs provide a lower bound on the number of spanning trees for all graphs on  $n$  nodes and  $\binom{k}{2} + c + (n - k - 1)$  edges, we seek a more straight-forward proof. In addition, these graphs have been conjectured to provide a lower bound on the All-Terminal Reliability when modeling certain kinds of networks. Our examinations involve application of Kirchhoff's Matrix-Tree Theorem and its consequences.

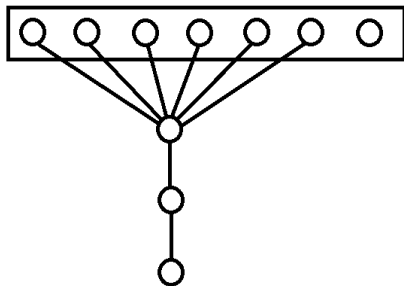


Figure: L Graph  $L(10; 7, 6)$  . Note that the rectangular box containing 7 nodes is a clique.

# L Graphs and related threshold and split graphs having a single bridge

Erin Lott, David Suazo, and Isabel Klingert

Our investigation involved L-graphs and Threshold and Split graphs in the same class having exactly one bridge. As a result of our investigation, our group has come up with a conjecture which predicts the multiplicity of Laplacian eigenvalues for L-graphs with  $k > c + 1$  and  $k = c + 1$  based on the degree sequence. A similar conjecture was found for split graphs. So far, we investigated graphs and their complements having clique order 5, 6, or 7 by finding their Laplacian eigenvalues.

# L Graphs and related threshold graphs

Francesca Hall, Dana Loughrey, Kristen Ralston

We have developed multiple examples of L-Graphs with at least two bridges and their threshold graphs from the same class. We aim to find a relationship between the Laplacian eigenvalues of both graphs. In doing so we have also examined the degree sequences, eigenvalues, spanning trees, majorizations and complements. We have constructed larger examples to confirm our observations.

# L Graphs Related to Split Graphs

Jennifer Bie, Dominic D'Antonio, David Guida, Arya Nabizadeh

We explored  $L(n,k,c)$  graphs with  $k = 5, 6,$  and  $7$  containing at least 2 bridges. Split Graphs were generated having the same  $k$  and class as traditional L Graphs for comparison. Our investigation led to two conjectures:

- 1 Given the degree sequence  $d_1, \dots, d_n = \Delta$  where  $d_i \leq d_{i+1}$  and the eigenvalues  $\lambda_1, \dots, \lambda_n$  where  $\lambda_i \leq \lambda_{i+1}$  then  $\lambda_n < \Delta_n + 2$  for both the L Graphs and the Split Graphs.
- 2 Given L Graphs and the corresponding Split Graph, when  $k \geq c + 1$ , there are  $(c-1)$  Laplacian Eigenvalues equal to  $(k+1)$ .  
Additionally, our graphs also satisfy the conjectures that other other groups established.